On-Wafer Noise Parameter Measurements

using

Cold-Noise Source

and

Automatic Receiver Calibration
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Abstract
Noise figure measurement accuracy is determined by the precision of the test source impedance, the DUT S-Parameter and the overall system calibration. A new measurement setup, based on the ‘Cold-Noise’ technique is presented which allows for in-situ recalibration of the noise figure meter without removing the DUT. Therefore, measurement errors due to drift in the receiver chain (LNA, mixer, filter and noise figure meter) can be eliminated.

Introduction
The two mostly used techniques for noise figure measurements include:

- **the standard technique**[1]; a noise source, connected to the input of the DUT is set to its ‘cold’($T_C$) and ‘hot’($T_H$) state, the respective noise power outputs are measured and used to calculate the noise figure of the DUT [1]. This technique is mostly used to measure the noise figure in a 50 Ohm system. By inserting a tuner in between the noise source and the

![Diagram of Noise Figure Measurement Setup](image)

**Figure 1: Noise Figure Measurement Setup**
DUT, noise figure measurements for source impedance different than 50 Ohm may be performed; this is required for the extraction of noise parameters of the device. However, the loss of the tuner must be known with high accuracy since it enters directly into the measured noise figure value. Tuners exhibit higher loss with increasing reflection coefficient. The S-Parameter of the DUT must be known to calculate the mismatch correction factor M [2].
• the ‘cold source’ measurement technique, first proposed by Adamian and Uhrl [3][4]. A noise source is only required during the calibration phase, to determine the kBG constant of

Reference Planes

Tuner
Noise Source

DUT
LNA Noise Figure Meter

Figure 2: Cold Source Measurement Technique

the receiver. A one-port tuner to adjust the source impedance is connected to the input of the DUT during noise figure measurements. The thermal noise of the tuner, proportional to the ambient temperature and the resistive part of its impedance, amplified by the DUT and the additional noise power generated inside the DUT are detected by the receiver. The noise figure of the DUT is calculated by knowing the source impedance, the S-parameter of the DUT and the input reflection coefficient of the receiver. This technique is most often used in automated noise parameter measurement systems.

Both of the above described techniques require a calibration to be performed prior to measurement start in order to characterize the receiver:

• Using the standard setup requires the noise source to be connected to the output reference plane, i.e. the input of the receiver, for calibration.

• The ‘cold source’ setup is calibrated by inserting a THRU connection instead of the DUT and measuring the ‘kBG’ constant of the receiver as a function of frequency. However, a modified ENR table for the noise source must be used in order to take into account the switch and other passive components inserted between noise source and DUT, thus the calibration reference is shifted to the input of the DUT. No connections have to be broken in the ‘cold-noise’ setup to perform a calibration when a switch is included.

Both techniques require different connections during calibration and measurement execution, either repositioning of the noise source, or replacement of the DUT. Especially during measurement sessions of long duration, i.e. probing an entire wafer, it is of great interest to recalibrate the setup to minimize errors caused by drift in the receiver components.
The proposed modified setup uses the cold noise technique and has the advantage of re-calibration without reconfiguration. The noise source is connected via a switch located in the output of the DUT to the noise figure meter during calibration. Since the S-parameter of the switch, for both positions, have been measured previously, the reference plane is shifted to the output of the DUT. The one-port input tuner, required to control the source impedance, is connected directly to the input port of the DUT.

The output switch, controlled by the measurement software, is set periodically to the calibration position, and the kBG constant of the receiver is re-calibrated; all other components of the setup are passive, therefore do not need any periodic verification.

**Figure 3: Modified Cold Source Setup**

![Diagram of the modified cold source setup](image-url)
Focus Microwaves

Test Setup Calibration

The basic setup shown in figure 3 has been extended to allow for on-wafer calibration and simultaneous measurement of noise figure and S-parameter, a SPDT switch was added in the input, the output switch was replaced by a transfer switch. Bias networks required to adjust the operation conditions of the DUT have also been added. The input tuner has been positioned as close as possible to the DUT, thus preserving a maximum reflection coefficient tuning range.
Prior to assembly of the setup, S-parameter measurements of the output transfer switch have to be executed for three positions: P1-P2, P1-P3, P4-P3. The P4-P3 data are required to transfer the ENR calibration of the Noise source to Port 3 of the switch. Also, two additional one port measurements are performed: $\Gamma_{NS}$, the reflection coefficient of the noise source, and $\Gamma_{RCVR}$, the input impedance of the receiver. These measurements are performed once, recalibration is required only after modifications to the setup.

To characterize the setup, the Vector Network Analyzer is first calibrated in the reference plane A-B using coaxial standards, preferably using TRL calibration technique. The quality of the calibration is very important since the tuner source impedance and the S-parameter of the DUT will be measured using the calibrated VNA. Then, using on-wafer TRL calibration standards (Thru, Delay and Reflect), the S-parameter of the input part (A-A') and the output part (B'-B) are determined. The tuner is set to its 'ZERO' position during the calibration, it acts like a matched transmission line with low loss. Using the previously measured S-parameter for the output transfer switch, the reference plane of the output network is shifted to port 3, where the receiver will be connected.
**Receiver Calibration**

The Receiver Calibration consists of two steps. First, the Gain-Bandwidth constant $k_{BG}$ has to be measured. This step is performed using the noise source, the output switch is set to connect the source (port 4) to the receiver (port 3).

Two power measurements are taken with the noise figure meter, $P_H$, (Source On) and $P_C$ (Source Off). The Gain-Bandwidth constant is calculated using [5]:

$$k_{BG} = \frac{P_H - P_C}{T_H - T_C} \left( \left| 1 - \Gamma_{RCVR} \Gamma_S \right|^2 \left( \left| 1 - \Gamma_S \Gamma_{NS} \right|^2 \left| S_{21} \right|^2 \right) \right)$$

with

$$T_H = T_0 \left( 1 + 10^{\frac{ENR(dB)}{10}} \right)$$

- $T_C$: actual temperature of source and receiver
- $T_0$: standard temperature (290K)
- ENR: Excess Noise Ratio of noise source
- $P_H$, $P_C$: measured Noise Power (Source On (SP9.2), Source Off (SP9.1))
- $\Gamma_S$: reflection coefficient seen by the receiver
- $\Gamma_{RCVR}$: reflection coefficient of receiver
- $\Gamma_{NS}$: reflection coefficient of Noise Source (OFF state)
- $S_{ii}$: S-parameter of network inserted between source and receiver

The measurement is repeated for all test frequencies.

The second step of the receiver calibration consists of determination of the noise parameter ($F_{MIN}$, $\Gamma_{OPT}$ and $R_N$) of the second-stage. Because of bandwidth considerations, no isolator has been included in the setup. Therefore, the noise figure of the receiver is dependent on the actual source impedance at its input. Having knowledge of noise parameter allows one to calculate the
actual noise figure of the receiver during noise figure measurements and to use highly accurate values for de-embedding of the second-stage noise contribution.

The input tuner is terminated in the 50\( \Omega \) load and a THRU substituted for the DUT. The tuner is set to a number (8...16) of settings, each presenting a different source impedance to the receiver. First, the impedance is measured, using the VNA, and then a noise power measurement is taken with the noise figure meter. The noise figure is calculated using [5]:

\[
F = \frac{P_C}{T_0 \cdot kBG} \left( \frac{1 - \Gamma_{RCVR}^2}{1 - |\Gamma_S|^2} \right) - \frac{T_C}{T_0} + 1
\]

with

- \( P_C \): Noise Power
- \( \Gamma_S \): source reflection coefficient
- \( \Gamma_{RCVR} \): input reflection coefficient of receiver
- \( kBG \): gain-bandwidth constant of receiver
- \( T_C \): actual temperature
- \( T_0 \): standard temperature (290K)

The noise parameter are calculated using Lane’s extraction technique [6], based on the noise figure measurements for different source impedances.

Having determined the gain-bandwidth constant \( kBG \) and the four noise parameter (\( F_{MIN} \), \( \Gamma_{OPT} \) and \( R_N \)) of the receiver, the system is fully calibrated for noise figure measurements.

A typical receiver calibration data set is shown below; for each frequency the RF attenuation setting, the minimum noise figure, the equivalent noise resistance, the optimum source admittance (real and imaginary part) and the gain-bandwidth constant are given.

<table>
<thead>
<tr>
<th>RF Attenuation Setting</th>
<th>( F_{MIN} )</th>
<th>( \Gamma_{OPT} )</th>
<th>( R_N )</th>
<th>kBG</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.000</td>
<td>+20</td>
<td>19.914</td>
<td>0.01821</td>
<td>0.59456135</td>
</tr>
<tr>
<td>6.000</td>
<td>+20</td>
<td>28.140</td>
<td>0.01237</td>
<td>0.76682953</td>
</tr>
<tr>
<td>8.000</td>
<td>+20</td>
<td>37.229</td>
<td>0.01106</td>
<td>0.48827391</td>
</tr>
<tr>
<td>10.000</td>
<td>+20</td>
<td>14.929</td>
<td>0.03009</td>
<td>0.44958811</td>
</tr>
<tr>
<td>12.000</td>
<td>+20</td>
<td>17.399</td>
<td>0.02107</td>
<td>0.47471935</td>
</tr>
<tr>
<td>14.000</td>
<td>+20</td>
<td>14.929</td>
<td>0.03009</td>
<td>0.44958811</td>
</tr>
<tr>
<td>16.000</td>
<td>+20</td>
<td>17.399</td>
<td>0.02107</td>
<td>0.47471935</td>
</tr>
<tr>
<td>18.000</td>
<td>+20</td>
<td>12.437</td>
<td>0.02565</td>
<td>0.66590498</td>
</tr>
</tbody>
</table>

These values are used during noise figure measurements to determine the actual noise figure of the receiver, which is function of its source impedance and will change as a DUT is inserted and the input is tuned to different impedances.
Source Impedance Pattern Calibration

Noise figure measurements for at least four different source impedances are required to solve for the four noise parameter, but more samples are advantageous in order to minimize the effect of measurement errors and allow averaging [7]. It has been shown that increasing the number of intermediate points does not improve the fitting results significantly, but a proper selection of the source impedances is beneficial [4]. M. Sannino recommends to measure at least 7 points, including one point located near the expected optimum source impedance, one point presenting 50 Ω to the device, and the remaining points located at intermediate values between 50 Ω and the $\Gamma_{OPT}$ location. The source impedances selected will be saved in a “pattern file”, thus allowing the test system to automatically repeat the measurement sequence using precisely defined positions.

In order to reduce measurement errors introduced by inaccurate source impedance values, the setup shall be recalibrated. A THRU has to be inserted in the DUT reference plane. The input tuner is set to the positions selected in the pattern file and a ‘one-port’ s-parameter measurement is triggered to determine the source impedance presented to the DUT for the given tuner position.

The following figure shows the location of source impedances used to perform noise figure measurements and subsequent noise parameter extraction.

![Figure 7: Source Impedance Pattern Definition](image-url)
Noise Figure Measurement and Noise Parameter Extraction

The noise figure of the DUT cannot be measured directly, but the noise figure of the DUT, OUTPUT NETWORK and RECEIVER is given by:

\[
F_{TOT} = \frac{P_i}{T_0 kBG} \frac{|1 - S_{11}\Gamma_i|^2 |1 - \Gamma_{RCVR}\Gamma_S|^2}{\left(1 - |\Gamma_i|^2\right)|S_{21}|^2} - \frac{T_c}{T_0} + 1
\]  
(eq. 3.01)

with

- \(P_i\): Noise Power
- \(\Gamma_i\): source reflection coefficient (seen by DUT)
- \(\Gamma_S\): source reflection coefficient (seen by RECEIVER)
- \(\Gamma_{RCVR}\): input reflection coefficient of receiver
- \(kBG\): gain-bandwidth constant of receiver
- \(T_c\): actual temperature
- \(T_0\): standard temperature (290K)
- \(S_{ii}\): S-parameter of DUT

Using

\[
F_{TOT} = F_{DUT} + \frac{F_{OUT} - 1}{G_{DUT}} + \frac{F_{RCVR}(\Gamma_S)_S - 1}{G_{DUT}G_{OUT}}
\]  
(eq. 3.02)

and

\[
F_{OUT} = \frac{1}{G_{OUT}}
\]  
(eq. 3.03)

where

- \(G_{DUT}\): available gain of DUT
- \(G_{OUT}\): available gain of output network (bias and switch)
- \(F_{RCVR}\): noise figure of Receiver for actual source impedance

the noise figure of the DUT is calculated:

\[
F_{DUT} = F_{TOT} - \frac{F_{RCVR}(\Gamma_S)S - G_{OUT}}{G_{DUT}G_{OUT}}
\]  
(eq. 3.04)

Noise figure measurements are performed either at manually selected source impedances, or in an automatic way by using predefined and in-situ calibrated points stored in a pattern definition.
The following figure shows the noise figure values measured for different source impedance values. A pattern file was used to tune automatically to the pre-calibrated points and trigger the measurement.

**Figure 8: Noise Figure Measurement Results**
Determination of Device Noise Parameter

The noise behavior of an active device is fully determined by its four noise parameters, and the noise figure of a device is defined as a function of the source admittance as:

\[ F = F_{\text{MIN}} + \frac{R_N}{G_S} |Y_S - Y_{\text{OPT}}|^2 \]  

(eq. 3.10)

where

- \( F_{\text{MIN}} \): minimum noise figure
- \( R_N \): equivalent noise resistance
- \( Y_S \): source admittance seen by device (\( Y_S = G_S + jB_S \))
- \( Y_{\text{OPT}} \): optimum source admittance (\( Y_{\text{OPT}} = G_{\text{OPT}} + jB_{\text{OPT}} \))

The noise figure may also be calculated using reflection coefficients instead of admittance:

\[ F = F_{\text{MIN}} + \frac{4R_N}{Z_0} \frac{|\Gamma_S - \Gamma_{\text{OPT}}|^2}{\left| 1 + \Gamma_{\text{OPT}} \right|^2 \left( 1 - |\Gamma_S|^2 \right)} \]  

(eq. 3.11)

where

- \( F_{\text{MIN}} \): minimum noise figure
- \( R_N \): equivalent noise resistance
- \( \Gamma_S \): source reflection coefficient seen by device
- \( \Gamma_{\text{OPT}} \): optimum source admittance
- \( Z_0 \): characteristic impedance

Sometimes, \( R_n/Z_0 \) is given as the single parameter \( r_n \), called the normalized equivalent noise resistance.

In principle, four nonsingular measurements of noise figure from different source admittances will determine the four noise parameters.

Since experimental errors occur both in the measurement of the noise figure and in the measurement of the source admittance, it is of advantage to take additional measurements and perform statistical smoothing, a procedure first proposed by R. Lane[6].
First, equation 3.10 has to be re-written in a form that is linear with respect to the four new parameter A, B, C, and D [9]:

\[
F = A + BG + \frac{C + BB^2 + DB}{G} 
\]  
(eq. 3.12)

where

- \( F_{MIN} = A + \sqrt{4BC - D^2} \)  
  (eq. 3.13)
- \( R_N = B \)  
  (eq. 3.14)
- \( G_{OPT} = \frac{\sqrt{4BC - D^2}}{2B} \)  
  (eq. 3.15)
- \( B_{OPT} = -\frac{D}{2B} \)  
  (eq. 3.16)

Since more than the four measurement points required will be used to solve the above equations, a least square fit taking into account additional measurement data is performed, using the following error criterion:

\[
\varepsilon = \frac{1}{2} \sum_{i=1}^{n} \left[ A + B \left( G_i + \frac{B_i^2}{G_i} \right) + C + \frac{DB_i}{G_i} - F_i \right]^2 
\]  
(eq. 3.17)

where

- \( F_i \): measured noise figure
- \( G_i + jB_i \): source admittance of \( i \)-th measurement point

The error criterion \( \varepsilon \) is minimized by building a system of linear equations using:

\[
\frac{\partial \varepsilon}{\partial A} = \sum_{i=0}^{n} \frac{1}{2} P = 0 
\]  
(eq. 3.18)

\[
\frac{\partial \varepsilon}{\partial B} = \sum_{i=0}^{n} \left( G_i + \frac{B_i^2}{G_i} \right) P = 0 
\]  
(eq. 3.19)

\[
\frac{\partial \varepsilon}{\partial C} = \sum_{i=0}^{n} \frac{1}{G_i} P = 0 
\]  
(eq. 3.20)

\[
\frac{\partial \varepsilon}{\partial D} = \sum_{i=0}^{n} \frac{B_i}{G_i} P = 0 
\]  
(eq. 3.21)

with
\[ P = A + B \left( \frac{B^2}{G_i} \right) + \frac{C}{G_i} + \frac{DB}{G_i} - F_i. \]

Equations 3.18 to 3.21 are solved for the unknowns A, B, C and D. Finally, the four noise parameter are calculated using equations 3.13 to 3.16.

The following figure shows the result of a noise parameter extraction: 11 noise figure measurements were taken at different source impedances and the four noise parameter were determined. In order to verify the accuracy of the noise parameter, the test software shows the measured and the calculated noise figure, based on the noise parameter and the source impedances. The last column shows the difference between the measured and the calculated value.

![Figure 9: Noise Parameter Extraction](image)
Noise Figure Measurement Example

S-parameter and noise figure measurement have been performed on a 600 µm GaAs MESFET device. The device has been biased at $V_d=2V$ with a drain current of 20mA. A Hewlett-Packard HP8510B network analyzer has been used. The following figures show the measured input and output reflection coefficient and the forward and reverse gain.
Noise measurements were taken at 4, 8, 12 and 18 GHz.

Figure 10: Measured S-Parameter (2-18GHz)

Figure 10: Measured Forward Gain (2-18GHz)
The measured minimum noise figure and the equivalent noise resistance are shown in the following figure.

![Figure 11: Measured Fmin and Rn](image)

**Figure 11: Measured Fmin and Rn**

The measured Optimum Source Impedance is shown below.

![Figure 12: Measured Optimum Source Reflection Coefficient](image)

**Figure 12: Measured Optimum Source Reflection Coefficient**

The test results in tabular format are shown below:
The last column shows the noise figure for a 50 Ohm source impedance.

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Fmin (dB)</th>
<th>Rn (dB)</th>
<th>MAG (dB)</th>
<th>ANG (°)</th>
<th>NF (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.000</td>
<td>0.41100</td>
<td>28.5000</td>
<td>0.72000</td>
<td>34.6500</td>
<td>1.86542</td>
</tr>
<tr>
<td>5.000</td>
<td>0.42139</td>
<td>26.6719</td>
<td>0.70623</td>
<td>41.4000</td>
<td>1.81240</td>
</tr>
<tr>
<td>6.000</td>
<td>0.45528</td>
<td>25.0625</td>
<td>0.68697</td>
<td>48.2864</td>
<td>1.78133</td>
</tr>
<tr>
<td>7.000</td>
<td>0.51211</td>
<td>23.6719</td>
<td>0.66423</td>
<td>55.2975</td>
<td>1.77643</td>
</tr>
<tr>
<td>8.000</td>
<td>0.59100</td>
<td>22.5000</td>
<td>0.64000</td>
<td>62.4200</td>
<td>1.80128</td>
</tr>
<tr>
<td>9.000</td>
<td>0.71989</td>
<td>21.7875</td>
<td>0.61674</td>
<td>70.4422</td>
<td>1.90328</td>
</tr>
<tr>
<td>10.000</td>
<td>0.84769</td>
<td>21.1333</td>
<td>0.59558</td>
<td>77.8943</td>
<td>2.01206</td>
</tr>
<tr>
<td>11.000</td>
<td>0.97440</td>
<td>20.5375</td>
<td>0.57666</td>
<td>84.8457</td>
<td>2.12709</td>
</tr>
<tr>
<td>12.000</td>
<td>1.11321</td>
<td>20.0231</td>
<td>0.56378</td>
<td>91.3612</td>
<td>2.24821</td>
</tr>
<tr>
<td>13.000</td>
<td>1.22448</td>
<td>19.5208</td>
<td>0.54560</td>
<td>97.4921</td>
<td>2.37559</td>
</tr>
<tr>
<td>14.000</td>
<td>1.34784</td>
<td>19.1000</td>
<td>0.53343</td>
<td>103.287</td>
<td>2.50964</td>
</tr>
<tr>
<td>15.000</td>
<td>1.47007</td>
<td>18.7375</td>
<td>0.52345</td>
<td>108.779</td>
<td>2.65096</td>
</tr>
<tr>
<td>16.000</td>
<td>1.59118</td>
<td>18.4333</td>
<td>0.51559</td>
<td>113.993</td>
<td>2.80030</td>
</tr>
<tr>
<td>17.000</td>
<td>1.71115</td>
<td>18.1875</td>
<td>0.50980</td>
<td>118.947</td>
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<tr>
<td>18.000</td>
<td>1.83000</td>
<td>18.0000</td>
<td>0.50600</td>
<td>123.650</td>
<td>3.12672</td>
</tr>
</tbody>
</table>

Figure 13: On Wafer Noise Measurement Setup 6-40 GHz
Figure 14: Millimeterwave On-Wafer Noise Measurement Setup
Conclusion

A novel noise figure measurement setup, using the cold source technique, has been presented. A calibration procedure for on-wafer measurement of low noise devices has been successfully implemented. The four noise parameter of the receiver allow the determination of the noise figure of the receiver during measurements, where the input of the setup is tuned using a precision mechanical tuner. In-situ S-parameter measurement of the DUT have been performed.

Noise figure measurements have been performed on 600µm GaAs MESFET device and results are given, including the noise parameter of the device.

Figure 15: On-Wafer Measurement Setup 0.8-18 GHz
REFERENCES

[1] Fundamentals of RF and Microwave Noise Figure Measurements, Hewlett Packard Application Note 57-1, July 1983

[2] Noise Measurements Using the Computer Controlled Microwave Tuner System, Focus Microwaves Application Note 1-90


